On the Formula for Black Body Radiation.

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In a recent paper* I showed that the experimental data with regard to "black body" radiation could be represented exceedingly well by an empirical formula for the radiation function at temperature θ, and wave-length λ, viz.:—

$$\phi(\theta, \lambda) = k\theta^5 \left\{ \frac{\lambda\theta}{\lambda^2\theta^2 + a^2} \right\}^4.$$

I further indicated an electric system which gives rise to an emission function which is a generalised form of the above empirical expression.

It is a matter of interest to see if one can learn more as to the character of a system which would give the empirical form precisely. The method of doing so by Fourier analysis was indicated by Lord Rayleigh.†

The form of the primary disturbance which leads to the above spectral distribution of energy is found to be very simple, and almost suggests itself.

If we take a function ψ of the time t such that

$$\begin{aligned} \psi(t) &= te^{-n_0 t} \quad \text{for} \quad \infty > t > 0, \\ &= te^{n_0 t} \quad \text{for} \quad 0 > t > -\infty, \end{aligned}$$

then the Fourier representation of the function is

$$\psi(t) = \frac{4n_0}{\pi} \int_0^\infty \frac{n \sin nt}{(n^2 + n_0^2)^2} \delta n,$$

and, further, for a number of arbitrary disturbances of this type we get

$$\int_{-\infty}^{+\infty} \{\psi(t)\}^2 \, dt = \frac{16 n_0^2}{\pi} \int_0^\infty \frac{n^2}{(n^2 + n_0^2)^4} \delta n.$$

The spectral distribution of $\{\psi(t)\}^2$ is thus of the form $\frac{n_0^2 n^2}{(n^2 + n_0^2)^4} \delta n$, and since n varies as λ^{-1}, we thus get the empirical form as far as λ enters.

The further interpretation of the formula now depends, as regards detail, upon whether we proceed in terms of pure dynamics or electro-dynamics. In the first case we should regard $\psi(t)$ as proportional to the velocity of displacement, and in the second as proportional to the acceleration of the

* 'Roy. Soc. Proc.,' A, vol. 89, p. 393 (1913).

† 'Phil. Mag.,' vol. 27, p. 460 (1889).

displacement. But in either case the radiation between frequency n and $n + \delta n$ will be expressed by

$$mA^2 \frac{n_0{}^2 n^2}{(n^2 + n_0{}^2)^4} \delta n,$$

where A is a quantity defining the amplitude of the disturbance, and m is the number of such disturbances per second.

In order to obtain Wien's displacement law n_0 must vary as θ, and then further we get Stefan's law if mA^2 varies as θ^7, and the formula reduces to the empirical form

$$k\theta^5 \left\{ \frac{\lambda\theta}{\lambda^2\theta^2 + a^2} \right\}^4 \delta\lambda.$$

The important point we have obtained is that, in order to have this energy distribution, the primary type of disturbance required is a solution of the equation

$$\ddot{x} + 2n_0\dot{x} + n_0{}^2 x = 0,$$

which expresses the motion of an aperiodic dynamical system. Further, n_0 must be proportional to θ. But the present analysis does not indicate how this occurs, nor how m and A separately depend on temperature.

The primary type of disturbance can also arise as a solution of electrodynamic equations. Such an interpretation raises points of importance which I have not yet had opportunity to consider fully.

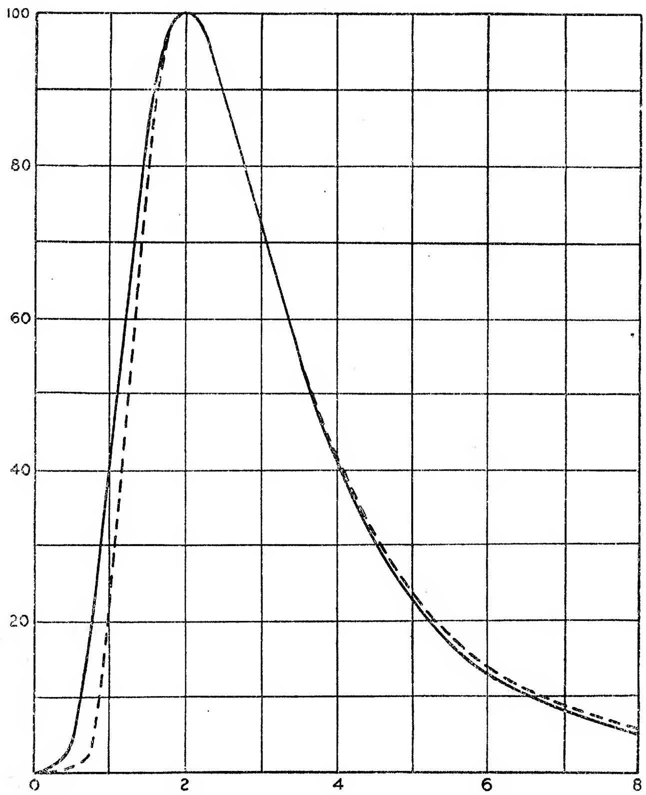
Appendix. [*Added January 29th*, 1914.]

In response to requests from several quarters as to the agreement between the formula and the data, I add the following information which I have been able to obtain.

If we choose the constants in Planck's formula and my own so that they give the same value for Wien's constant $a = \lambda_m\theta$ and so that they agree at the maximum, we get the following table:—

λ/λ_m.	Planck.	Walker.
0·0	0·0	0·0
0·25	0·0004	0·0490
0·333	0·0117	0·1292
0·5	0·2217	0·4096
0·666	0·6302	0·7260
1·0	1·0	1·0
2·0	0·4054	0·4096
3·0	0·1384	0·1292
4·0	0·0564	0·0490
10·0	0·0022	0·0015
∞	0·0	0·0

The two curves are shown in the figure.



Interrupted line—Planck's curve. Continuous line—Walker's curve.

The ordinates by Planck's formula are less than those by my formula for $\lambda/\lambda_m < 1$ and greater than mine when $\lambda/\lambda_m > 1$. But the area of the whole curves I find to be the same to 3 per cent.

Passing to the data, it is clearly not easy to discriminate between the formula within small ranges of wave-length. Thus Warburg's recent work,* within a range of about 1 μ, is not of much assistance. I have had to go back to the older measurements.

Lummer and Pringsheim† give a curve of emission for $\theta = 1650°$ absolute from $\lambda = 0·5\ \mu$ to about $18\ \mu$. I find that it lies almost exactly between the two theoretical curves, the advantage being very slightly in favour of my own formula.

Paschen‡ finds very good agreement with Planck's formula for any one wave-length at different temperatures; but he explains that in order to fit

* 'Ann. d. Phys.,' 1913, p. 613.

† 'Verh. d. Deutsch. Phys. Gesell.,' 1900, p. 177.

‡ 'Ann. d. Phys.,' vol. 4, p. 277 (1901).

Planck's formula over his range from about 1 μ to 9 μ he requires to assume that for "some unknown cause" the spectrum is weakened in passing to increasing wave-lengths, and that the observed values must be multiplied by increasing factors, the factor at 9 μ being 1·3, I understand, as compared with 1 at 1 μ. This is hardly convincing proof, and the facts suggest that my formula would avoid the necessity for these factors.

Rubens'* values for long waves up to 50 μ are relative and give good agreement with any formula that converges to Lord Rayleigh's form. If, however, we could compare the values for long waves with those at 1 μ, we have a means of deciding between the two formulæ as follows.

Writing Planck's formula in the form

$$k\,(4{\cdot}9651\,a)\,\lambda^{-5}/(e^{4\cdot9651\,a/\lambda\theta}-1),$$

we get the comparative table—

	Planck.	Walker.
Maximum occurs when	$\lambda_m\theta = a$	$\lambda_m\theta = a$
Total radiation is	$0{\cdot}0530\,k\theta^4a^{-3}$	$0{\cdot}0982\,k\theta^4a^{-3}$
Maximum radiation is	$0{\cdot}0348\,k\theta^5a^{-4}$	$0{\cdot}0625\,k\theta^5a^{-4}$
Value for long waves	$k\theta\lambda^{-4}$	$k\theta\lambda^{-4}$

Thus, if we chose k in each formula so as to get the total radiation or (as it happens) the maximum radiation to agree, then Planck's ordinates for long waves would be 1·8 times the ordinates from my formula.

It is worth while to note that it must be rather difficult to determine the maximum wave-length accurately. The table suggests that we could get a independently and hence λ_m for any temperature by comparing the total and the maximum radiation. Both formulæ would give the same value for a to 3 per cent.

* 'Ann. d. Phys.,' 1901, vol. 4, p. 649.